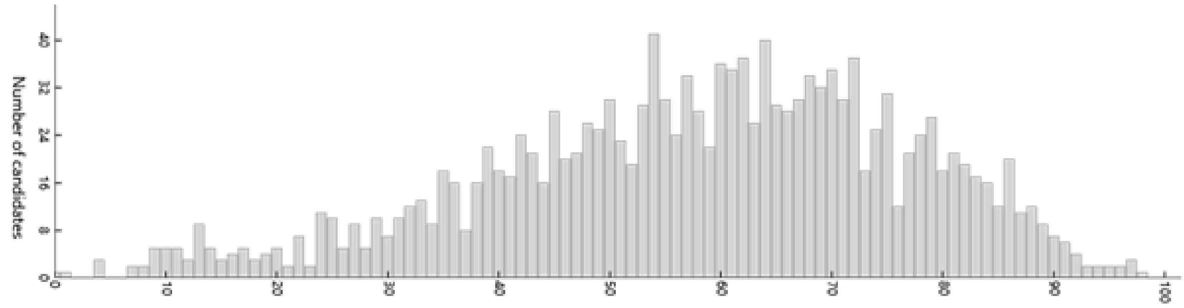
Government of **Western Australia**
School Curriculum and Standards Authority

2020 ATAR course examination report:
Mathematics Specialist

Year	Number who sat	Number of absentees
2020	1526	23
2019	1435	32
2018	1546	21
2017	1463	12

The number of candidates sitting and the number attempting each section of the examination can differ as a result of non-attempts across sections of the examination.

Examination score distribution–Written



Summary
The examination consisted of Section One: Calculator-free and Section Two: Calculator-assumed.

Attempted by 1525 candidates	Mean 57.74%	Max 97.82%	Min 0.00%

Section means were:			
Section One: Calculator-free	Mean 64.56%		
Attempted by 1525 candidates	Mean 22.59(/35)	Max 35.00	Min 0.00
Section Two: Calculator-assumed	Mean 54.08%		
Attempted by 1525 candidates	Mean 35.15(/65)	Max 64.24	Min 0.00

General comments
The paper appeared to be well received, yet it contained some different questions that required a deep level of understanding. However, the paper still contained a range of questions allowing the typical Mathematics Specialist candidate to show facility with key standard concepts.

A proportion of the candidates did not appear to be prepared for questions, such as Question 4, that involved the case of non-parallel planes yielding no solutions. It is notable that this concept had not been examined previously in the Mathematics Specialist ATAR papers of 2016–2019. Question 6 linked several separate areas of the course (inverse functions, implicit differentiation, and integration by partial fractions) and many candidates seemed unaccustomed to this combination of ideas.

Questions requiring concept development/proof (Questions 8 and 20) allowed the most capable candidates to exhibit their talent.

The length of the paper was deemed to be appropriate, as evidenced by the high percentage of candidates attempting Questions 19 (99%), 20 (92%) and 21 (91%).

The distribution of marks in 2020 exhibits a large spread, indicated by the standard deviation of 18.79%, which was similar to the 2019 figure of 18.93%. This points to the wide range of abilities of the Mathematics Specialist 2020 cohort.

Advice for candidates
- Ensure that working is copied correctly from line to line. For example, in Question 2, many candidates wrote a negative number in one line that then became a positive number in the next line.
- Improve the command of algebra and the use of brackets.
- Ensure your working shows an obvious sequence of steps and conclusion, enabling the marker to follow your line of thought and to observe a conclusion. Markers cannot be expected to extract meaning from a collection of numbers on the page.
- Ensure that the correct units are used in giving answers, particularly in questions asking for a rate of change. Careful reading of the question is central to this.
- Improve the legibility of digits.

Advice for teachers
- Provide opportunities for students to prove mathematics results, specifically in the vectors section of the syllabus.
- Improve the conceptual understanding of the intersection of planes in space, the understanding of the connection between logistic growth and its corresponding differential equation, and with motion where the velocity is given as a function of displacement.
- Emphasise the use of correct mathematics vocabulary and provide students with opportunities to explain mathematics concepts.

Comments on specific sections and questions
Section One: Calculator-free (49 Marks)
Candidates performed quite well on many straightforward questions in this section. The mean score of 64.56% reflected this. It was gratifying to see a number of able candidates being able to show concise and original solutions in Question 8.

Strengths in this section were:
- determining the rational function coefficients from the graph (Question 3)
- sketching the graph of the reciprocal of a given function (Question 5 part (a))
- writing the inverse rule of the given trigonometric function (Question 6 part (a))
- using an integration constant for an anti-derivative (Question 6 part (d)).

Weaknesses evident in this section were:
- calculation of the cross product (Question 2 part (a))
- understanding of the intersection of planes in space (Question 4)
- algebraic shortcomings with the integration using a given substitution (Question 7).

Question 1 attempted by 1511 candidates Mean 2.16(/3) Max 3 Min 0
This was a straightforward question to begin the paper, with most candidates scoring reasonably well. Substitution errors using the trigonometric identity were common as were evaluating correctly. Some candidates incorrectly thought that they could take a factor of 4 outside the integral expression.
Question 2 attempted by 1469 candidates Mean 3.92(/5) Max 5 Min 0
Pleasingly, a high number of candidates recognised that a cross product would determine the normal vector for this plane. Far too many candidates attempted to do the number work

‘in their heads’ without putting work to paper and so did not correctly determine the cross product. Part (b) was straightforward, yet many candidates could not correctly transcribe the normal vector found from part (a).

Question 3 attempted by 1518 candidates Mean 4.33(/6) Max 6 Min 0
Candidates performed very well with this familiar question. Justifications for answers were generally given using appropriate references to asymptotes and x intercepts. Some candidates mistakenly thought that $f(6)=3$. Those candidates who assumed this were given credit for obtaining $a=\frac{63}{32}$.

Question 4 attempted by 1515 candidates Mean 4.03(/7) Max 7 Min 0
In part (a), a wide variety of responses was provided, with many giving vague references to ‘planes are not multiples of each other’. The efficient response was to refer to the normal vectors and then make the appropriate conclusion that none of the planes were parallel. The solving of the system of equations in part (b) was supposed to be a straightforward question, but for many this proved problematic. A good number were not aware that their work pointed to there being no solution. Candidates often contradicted themselves in part (c) with what they had written in part (a) indicating confusion. Well-informed candidates drew a diagram showing the three planes intersecting in lines, and this was generally accepted. However, for full marks, candidates had to state that there was no common point of intersection. This part of the syllabus appears to need greater teaching emphasis.

Question 5 attempted by 1522 candidates Mean 4.79(/7) Max 7 Min 0
Candidates performed very well in this question. The drawing of $y=\frac{1}{f(x)}$ has been well taught and understood. In part (b), capable candidates were able to make the connection that $h(x)=2\times\frac{1}{f(x)}$. Those that did were able to write the range correctly. Candidates who used the notation $[2,\infty)$ for the set $y\geq 2$ had a more difficult task trying to express themselves correctly in this question.

Question 6 attempted by 1518 candidates Mean 7.95(/13) Max 13 Min 0
Part (a) was a straightforward task to determine the defining rule of the inverse trigonometric function. Candidates were not as confident in part (b) in differentiating implicitly a trigonometric function as expected. Those that did the implicit differentiation correctly often did not know how to proceed and obtain the result in terms of $x.$ In part (c), the coefficients q, r were well found using the suggested approach. Most did not show that the third equation/coefficients were consistent and so scored a maximum of two marks. The small minority of candidates that decided for themselves to use $\frac{px+q}{x^2+4}+\frac{r}{x-3}$ found that $p=0$ and hence were generally successful in scoring full marks. Failure to answer part (b) still enabled candidates to score three marks out of four for part (d). Most recognised that the natural logarithm of an absolute value function was required as was a constant of integration. A small number realised that the result from part (b) was required and so were able to write the inverse tangent function as an anti-derivative.

Question 7 attempted by 1518 candidates Mean 3.88(/5) Max 5 Min 0
This was a straightforward question in integration using a given substitution. However, it was disappointing to see how many candidates could not perform basic algebraic manipulation to simplify the integrand. The evaluation was also designed to be quite straightforward.

Question 8 attempted by 1259 candidates Mean 0.56(/3) Max 3 Min 0
A good number of candidates made an honest attempt to answer this question but were not able to detect the pattern with the sum of the first four terms. Markers looked for evidence of a logical sequence in the workings, and this enabled the most able candidates to stand out.

Section Two: Calculator-assumed (86 Marks)
Candidates did not perform as well (mean of 54.08%) compared to the Calculator-free section, primarily since it contained some questions that required a depth of conceptual understanding beyond routine skill or textbook-type questions. Question 15 part (b) and Question 20 were more testing for candidates. The last question on the paper (Question 21) was generally well attempted, despite many bypassing or finding Question 20 too difficult. Most candidates were able to construct a fair response to this question that tested understanding of how a function's graph determines the number of solutions to an equation.

Strengths in this section were:
- solving the complex equation giving solutions in correct polar form (Question 13)
- the improved performance in making a comparison between a sample mean and a known population mean (Question 18 part (c))
- use of the technique of increments (Question 19 part (c)).

Weaknesses evident in this section were:
- choosing the appropriate form of a complex number (Question 11 part (a))
- converting a complex number (in quadrant III) into the correct polar form (Question 15 part (a))
- forming the correct area expression using an appropriate definite integral (Question 16 part (a))
- the ability to form correct vector expressions (Question 20 part (a))
- the ability to form the correct area expression (Question 20 part (b)).

Question 9 attempted by 1491 candidates Mean 2.69(/4) Max 4 Min 0
Candidates performed well with writing the correct expression to determine the full volume. Those lacking competence in algebra thought that $\left(\sin\left(\frac{y}{\pi}\right)+3\right)^2=\sin^2\left(\frac{y}{\pi}\right)+9$, while others wrote $x=\sin\left(\frac{y}{\pi}\right)-3$. Common errors were forgetting to include a factor of π, having their CAS calculators set in degree mode or not being able to correctly code in the intended expression. However, for many this was a good start to the Calculator-assumed section, where it was essential to use the CAS calculator appropriately.

Question 10 attempted by 1484 candidates Mean 2.52(/7) Max 7 Min 0
Part (a) had an obvious element to it (the modulus equation), and yet a more subtle element (the correct argument inequality). Candidates need to be careful that when they write the argument of a complex number they are cognisant of the point from which the argument is measured. Several outstanding candidates wrote $0 \le Arg\left(z-2i\right) \le \frac{\pi}{2}$, which made the task easier to express the argument inequality. The type of locus required in part (b) had not previously been examined in 2016–2019 and this was reflected in candidates' responses, despite this type of question appearing in textbooks. Candidates should try to interpret the given statement in terms of distances i.e. one distance is $\sqrt{5}$ more than another distance.

Question 11 attempted by 1457 candidates Mean 2.92(/5) Max 5 Min 0
This was anticipated to be a straightforward question but it was not to be for this cohort. Far too many candidates chose to work in Cartesian form rather than work with polar form, and this inevitably led to errors. Poorly presented/illegible work was evident here. Candidates performed slightly better in part (b) but given that they had access to a CAS calculator, it was difficult to fathom many of the responses.

Question 12 attempted by 1492 candidates Mean 4.34(/6) Max 6 Min 0
A good majority of candidates knew that they needed to determine the initial z coordinate on the three-dimensional curve. Part (b) was straightforward and well done. A number of candidates erroneously thought by showing that the z component of the acceleration vector was zero that this would indicate constant velocity for part (c). Candidates needed to show that the speed was independent of time, so those that invoked the Pythagorean identity generally scored full marks. Some others opted to calculate the distance travelled and so found the average speed, which was not what the question required.

Question 13 attempted by 1498 candidates Mean 3.48(/4) Max 4 Min 0
Candidate performance was good on this routine question in solving a complex equation using de Moivre's Theorem. The common errors were not determining the correct modulus or argument of the right hand side of the equation. Virtually all candidates used the correct convention $-\pi < \theta \le \pi$, which was pleasing.

Question 14 attempted by 1510 candidates Mean 3.15(/5) Max 5 Min 0
Candidates continue to experience difficulty dealing with rectilinear motion where the velocity is given as a function of position. Some of the difficulty appears to lie with careless use of notation or lack of detail about which variable the differentiation relates to. Performances were disappointing on part (b), essentially testing candidates' knowledge of the condition for simple harmonic motion. Only the more assured candidates fared well here. In part (c), candidates needed to express the given velocity statement $v=-0.2x$ as $\frac{dx}{dt}=-0.2x$, and then realise an exponential function for $x\left(t\right)$ would be required.

Question 15 attempted by 1454 candidates Mean 2.36(/6) Max 6 Min 0
Candidates performed reasonably well in part (a), but it was not anticipated that so many would not be able to convert $-2(i+1)$ into the correct polar form, with many opting to write $-2cis\left(\frac{\pi}{4}\right)$, not realising that the radial coordinate is by definition/convention a positive value. Part (b) was one of the more difficult questions on the paper, but one that permitted able candidates to express themselves well. It was pleasing to see many begin by drawing a vector diagram rather than trying to use Cartesian form. However, many did not read the condition that $z = rcis\theta$ had to be drawn in the second quadrant since $\frac{\pi}{2} < \theta < \pi$ (which was stated on the opposite page). Many also opted to draw vectors but did not show that the magnitudes for ri and $rcis\theta$ were equal.

Question 16 attempted by 1445 candidates Mean 3.09(/5) Max 5 Min 0
It was somewhat surprising that so many candidates could not form a correct integral expression that would give the required area. Those writing the area expression as $\int_0^2 \sqrt{\sin\left(\frac{\pi x}{2}\right)+1}\, dx$ clearly did not understand what area/region this represented. In part (b), most realised that they needed to compare their answer in part (a) with the area of the circle having a radius of one unit. A common error was to believe that the radius was two units, meaning that they misread the graph scale.

Question 17 attempted by 1500 candidates Mean 5.11(/8) Max 8 Min 0
Part (a) was straightforward, and part (b) was well done. There were some candidates who were confused about not knowing the value for n, which was irrelevant for this question. Performance in part (c) was quite disappointing. Candidates needed to determine the connection between the standard deviations for the sample means from the different sample sizes. Many could not correctly express the required event $\left|\overline{X} - \mu\right| < 10$.

Question 18 attempted by 1488 candidates Mean 6.38(/11) Max 11 Min 0
A familiar question, part (a) was well done. Candidates need to realise that for a four mark question, details such as stating that the sample mean is normally distributed is a requirement. It was rather puzzling that many opted to use $\sigma\left(\overline{X}\right) = 1.5$, when it was clear that $\sigma = 1.5$ was the population standard deviation. There were disappointing performances in part (b), given that the question virtually stated that the margin of error had to be less than 0.2. Too many candidates wanted to rely on the formula sheet formula. Pleasingly though, almost all knew that they needed to conclude using an integer value. The last part was similar in format to the 2016 paper Question 19 part (e). Candidates seemed better prepared for this and many realised that some confidence interval was required. However, confusion centred on what comparison needed to be made. Clearly, the population mean for BikkiesAreUs was known ($\mu = 7.5$) and the population mean for YouBeautChokkies was not known, so a confidence interval for the YouBeautChokkies population mean had to be formed. Conclusions tended to be well written, with many stating that Charlie’s claim was not valid. Many candidates compared ‘overlapping’ confidence intervals. They should be taught that the relevant comparison is whether the BikkiesAreUs population mean falls within the confidence interval for the population mean for YouBeautChokkies.

Question 19 attempted by 1518 candidates Mean 6.42(/11) Max 11 Min 0
Some candidates did not read the question carefully enough i.e. $P(t)$ being measured in millions of tonnes. Correct units were required for full marks in part (a). Responses in part (b) were disappointing. Given that the form of the differential equation presented in the question was identical in format to that on the formula sheet, it indicated that many do not understand how the differential equation related to the given function $P(t)$. Candidates generally knew how to apply the increments concept in part (c), and many were able to achieve follow-through marks from the values obtained at part (b). Part (d) was done well by the more capable candidates. It was interesting to see many candidates avoiding part (e), indicating that the connection between the differential equation and the resulting graph of the function is not known. Only the highest scoring candidates could sufficiently justify why there would be no point of inflection seen in the graph.

Question 20 attempted by 1397 candidates Mean 2.49(/9) Max 9 Min 0
It appeared that this proof question was beyond the typical candidate of this cohort. Those that could write correct vector expressions for the sides could progress in this proof. A mark was awarded for correct notation and it was disappointing that so few could demonstrate correct notation. Performance on part (b) was much better than part (a), where candidates needed to form the correct expression for the area. Many realised that this was a related rates question and made a fair effort at this. Again, candidates must be encouraged to write correct units for any rate of change.

Question 21 attempted by 1391 candidates Mean 1.56(/5) Max 5 Min 0
A good number of candidates attempted this question. Some thought that all that was required was a sketch of the graph of $y = |f(x)|$. A handful of candidates explained or showed how they were finding solutions, and received a mark for this. The marking key proved to be highly effective in allowing candidates to score marks without completely sketching the required function. Many candidates did not consider the case where $k < 0$.